# Game-theoretic Understanding of Price Dynamics in Mobile Communication Services

Seung Min Yu and Seong-Lyun Kim

*Abstract*—In the mobile communication services, users wish to subscribe high quality service with a low price level, which leads to competition between mobile network operators (MNOs). The MNOs compete with each other by service prices after deciding the extent of investment to improve quality of service (QoS). Unfortunately, the theoretic backgrounds of price dynamics are not known to us, and as a result, effective network planning and regulative actions are hard to make in the competitive market. To explain this competition more detail, we formulate and solve an optimization problem applying the two-stage Cournot and Bertrand competition model. Consequently, we derive a price dynamics that the MNOs increase and decrease their service prices periodically, which completely explains the subsidy dynamics in the real world. Moving forward, to avoid this instability and inefficiency, we suggest a simple regulation rule which leads to a Pareto-optimal equilibrium point. Moreover, we suggest regulator's optimal actions corresponding to social objectives such as user welfare maximization or fairness between MNOs.

*Index Terms*—Network economics, game theory, competition, price dynamics, regulation, mobile communications.

## I. INTRODUCTION

### A. Conflict of Interests among Mobile Network Operators, Users, and the Regulator

In mobile communication services, there is interaction among *mobile network operators (MNOs)*, *users*, and *the regulator* (Figure 1). Each MNO makes an investment in its network to improve the quality of service (QoS) and sets a service price to maximize its profit. The users decide which MNO is more appropriate to subscribe the network service considering the service price and the QoS. Finally, the regulator aims to maximize the welfare of all users. Therefore, there should be some equilibrium points for the service price and network investment (QoS) between MNOs and users. Theoretically, finding such equilibrium points is not easy. The situation becomes even more complicated when there are multiple MNOs competing with each other.

Maximizing profit is the primary concern of MNOs, which might be achieved by having a high price level and low investment on the network. On the other hand, users wish to maximize their utility by consuming high QoS with a low service price. The QoS is directly related to the network investment from MNOs. Therefore, there is a conflict of interests among these players and the role of the regulator is very important. From the regulatory

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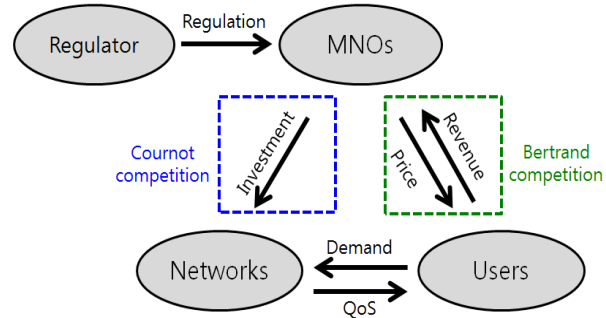


Fig. 1. The interaction among MNOs, users, and the regulator.

perspective, solely maximizing profit by MNOs should be avoided if it is at the cost of sacrificing user welfare.

For making efficient regulations, we firstly investigate characteristics of the competitive mobile communication services. An important question for MNOs is how much of the network capacity should be provisioned and how high the service price should be. Price competition between two operators was previously studied by Walrand [1], where the network capacity was assumed to be given. Here, we analyze how each MNO determines the optimal investment on the network and the service price as a response to the strategy of its competitor. For this purpose, we apply Cournot and Bertrand competition models [2]-[5].

In the Cournot model, MNOs compete with each other deciding the extent of investment on their networks. On the other hand, in the Bertrand model, MNOs engage in price competition to attract more subscribers for a given network capacity. We combine the Cournot and Bertrand models so that the network capacity is determined in the Cournot phase and afterwards the service price is determined in the Bertrand phase. The Cournot and Bertrand models are interlinked and we achieve joint optimization of the network capacity and service price. Our main viewpoint of this joint optimization is in investigating the dynamics of competition between MNOs and also in finding an optimal role of the regulator.

### B. Price Competition and Subsidization

The dynamics of price competition among network operators was studied in some previous works [6]-[10]. Particularly, in [6], we showed that there would be a price dynamics that network operators increase and decrease their service prices periodically. In the real world, however, network operators' billing systems are very similar in the
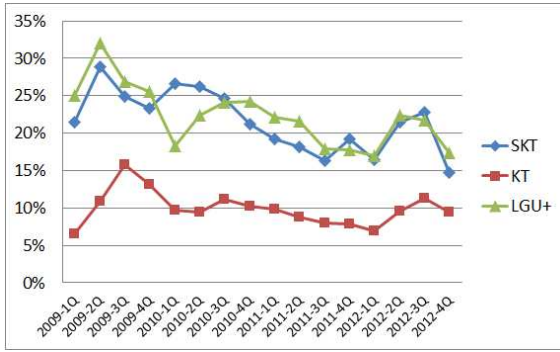
Fig. 2. Quarterly marketing expenses as percentage of sales of the major MNOs (SKT, KT and LGU+) in South Korea.

same country or state and the price dynamics does not seem to occur. Moreover, the previous works did not give a clear explanation for the reality of the price dynamics.

For discussing the reality of the price dynamics, let us consider the monthly charge structure in the mobile communication service. In most countries, many MNOs give a *subsidy* to attract new subscribers (potential users or their competitors' subscribers) [11]. The subsidy is offered as part of a contract that includes a stipulated time period. Therefore, we should consider not only the initial service price but also the subsidy amount for examining the price competition among MNOs.

To show that there can be a kind of price dynamics by subsidization in the real world, we plot quarterly marketing expenses as percentage of sales of the major MNOs (SKT, KT and LGU+) in South Korea as an example (Figure 2). Note that the investigated marketing expenses exclude advertising expenses, which are almost used for subsidization. In the figure, the MNOs increase and decrease their marketing expenses repeatedly, and this can be interpreted as the service price dynamics by subsidization. Then, why do the MNOs use subsidization as an indirect method for increasing or decreasing their service prices? The MNOs cannot increase the service prices easily due to regulations. On the other hand, there are few regulations on subsidization and people are relatively generous about change of the subsidy amount because it is known that subsidization is essentially a means of lowering the cost of new subscriber's entry to the mobile communication services [12]. Therefore, the MNOs compete with each other by adjusting their subsidy amounts and there is the price dynamics by subsidization in the real world. Unfortunately, the theoretic backgrounds of price dynamics are not known to us, and as a result, effective network planning and regulative actions are hard to make in the competitive market.

In this paper, we *theoretically explain* the price dynamics between MNOs using the two stage competition model, where the MNOs increase and decrease their service prices periodically without an equilibrium point. This kind of price dynamics is not desirable to any players due to the instability. For example, it is unfair that users' payments for the mobile communication service are different in different start times of the subscription even though they

are served by the same MNO. Based on our analysis, to avoid such instability, we suggest a simple regulation rule that makes an equilibrium point of price levels, which is Pareto-optimal.

### C. Related Work

In [13], the author shows that service price and QoS are inter-related in communications networks, and suggests Paris metro pricing (PMP) for Internet. PMP is a kind of price discrimination over different QoS levels; the higher QoS, the higher price. In that paper, the author finds that the service price and QoS will converge to an equilibrium point after a number of interactions. PMP is further extended by Walrand [1], who formulates an Internet pricing model under price and QoS constraints. In that work, the author investigates how much PMP improves the operator's profit compared to a single optimal service price. The author also analyzes price competition between two homogeneous network operators, the network capacities of which are fixed. In [6], we show the dynamics of price competition (*price war*) using the Walrand model [1], and suggest a regulation for price level convergence.

The price war in communication service is observed in [6]-[10]. Particularly, in [7] and [8], if one operator lowers its price to increase revenue or to monopolize the entire market, then the other operators will also lower their price to match the price leader. The price down competition will occur repeatedly among all operators, eventually damaging every operator with a revenue decrease.

Competition among network operators occurs not only by price differentiation. The capacity of the network is another important variable. This is because users will select a network operator based on decision criteria including not only service price but also QoS level, and the QoS is directly related to the network capacity. Therefore, each operator jointly optimizes the service price and network capacity. All of the previous work mentioned above focuses only on price competition, assuming the network capacity is given as an external value. In [14], the authors consider competition among multiple network operators with single- or two-service classes. In that work, service prices are fixed and price competition does not occur. To attract more users, the operators decide only the network capacity.

Suppliers of a homogeneous good/service compete with each other by deciding their amount of output. This is called Cournot competition (quantity competition) [4]. Generally, the market price decreases as the total amount of output increases. On the other hand, Bertrand competition refers to price competition where the suppliers compete with each other by controlling the product price [5]. In the Bertrand competition model, consumers buy all of a particular product from the supplier with the lowest price.

### D. Main Contribution of This Paper

Using the two stage model [15], we will show that two MNOs initially decrease their service prices (i.e., increase subsidies) as in [7] and [8], but one MNO suddenly

increases its price when the competitor's price is lower than a certain threshold. Therefore, the price levels increase and decrease periodically without an equilibrium point. We call this *price war with a long jump*, which is not desirable to any players.

The main contributions and results of this paper are summarized below.

- *Description of price dynamics*: In the real world, MNOs tend to compete with each other changing their service prices by subsidization (see Figure 2). Using a two-stage Cournot and Bertrand competition model with network congestion, we mathematically analyze the competition between MNOs. Based on the game-theoretic approach, we show that there exists price (subsidy) dynamics in the mobile communication service, which well explains the subsidy dynamics in Figure 2.
- *Regulation for price convergence*: To avoid instability and inefficiency, we propose a simple regulation that limits the number of price level changes. We show that the regulation makes an equilibrium point of price levels, which is Pareto-optimal. We also introduce more realistic regulations that bring the same effect of the price regulation.
- *Regulation under a two-stage Cournot and Bertand model*: Using the two-stage model, we calculate an equilibrium point of the network capacity and the service price. From the result, we suggest a regulator's optimal action (exacting taxes) corresponding to the objectives such as user welfare maximization or fairness between MNOs. This is an extension of our previous work [6] to the two-stage model.

The rest of this paper is organized as follows. In the next section, we describe our system model. Section III presents our optimization problem in two-stage duopoly competition. In Section IV, we derive a solution to the optimization problem in the Bertrand stage and explain how two operators' prices vary. We suggest a simple regulation that drives the price levels to converge on an equilibrium point. In Section V, we combine the Bertrand model with the Cournot model. Using a backward induction method, we solve the optimization problem in the Cournot stage and find an equilibrium point. From the results, we describe characteristics of the communications service market and introduce the role of the regulator. Finally, Section VI concludes the paper.

## II. SYSTEM MODEL

Consider a service area covered by two competitive MNOs. There are $M$ users for the mobile communication service. Nonnegative values $k_1$ and $k_2$, respectively, denote the first and the second MNO's capacity, which determine the quality of service (QoS) of the networks. MNOs determine the optimal $k_1$ and $k_2$ values in the Cournot stage. In the Bertrand stage, MNOs compete by controlling $p_1$ and $p_2$, the first and the second MNO's price for the service. These service prices include the subsidy amounts (i.e., the

initial service price minus the subsidy amount). Therefore, the MNOs can control the service prices by adjusting the subsidy amounts even if there are some regulations that prohibit the initial price level changes. Without loss of generality, we assume $p_1$ and $p_2$ are normalized values over the interval $[0, 1]$. Each MNO can provide only one price to all users at a given time. The QoS of a network depends on the congestion level of the network. We denote the QoS of each MNO's network by $q_1$ and $q_2$, respectively. Without loss of generality, the values of $q_1$ and $q_2$ are also normalized over the interval $[0, 1]$. A value closer to 0 denotes a higher congestion level (lower QoS).

Each user decides whether to subscribe the communication service or not by selecting its serving MNO. Some users prefer high QoS (low congestion) even though they have to pay more. On the other hand, some other users will accept a low QoS if the service price is low, as was also noted by Paris metro pricing [13]. Willingness-to-pay and the QoS required by users are positively correlated. To model user behavior, we define the *user type* as in [1] and [14]. The user type $\alpha$ is a variable over $[0, 1]$ that quantifies the user's willingness-to-pay. At the same time, it quantifies the QoS level required by the user.[1] The value $\alpha$ is close to 1 when the user is willing to pay a high price for high QoS. At the other extreme ($\alpha \to 0$), the user prefers low QoS with a low price. Since it is difficult to figure out the user type value of each user, we assume that it is a random variable (e.g., uniform distribution in $[0, 1]$).

Consider a user with user type value $\alpha$. For the user to subscribe the communication service offered by MNO $i$, both the price and QoS levels should be satisfied. In other words, $\alpha \geq p_i$ and $\alpha \leq q_i$, where we regard the first and second inequalities as the *price condition* (PC) and *QoS condition* (QC), respectively.[2] The PC is commonly used in microeconomics [16], but it is not sufficient to model communication service, where some users whose PCs are satisfied may not use the service because the QoS is lower than expected due to congestion. If multiple MNOs satisfy both conditions, then the user will select the MNO offering the lowest price.

We use a linearly decreasing QoS model as in [1] and [14], which mirrors the perception of service quality [17]:

$$q_1 = 1 - \frac{d_1}{k_1 M}, \quad q_2 = 1 - \frac{d_2}{k_2 M}, \tag{1}$$

where $d_1$ and $d_2$ denote the number of users accessing the first and the second MNO, respectively. We define the

[1]The user type $\alpha$ has dual role as willingness-to-pay and QoS requirement. This seems to be open to dispute because those two criteria cannot merged into one-dimensional parameter space. In this paper, however, we assume that each user's willingness-to-pay and QoS requirement are highly correlated and can be modeled by one-dimensional parameter for the mathematical tractability.

[2]The characteristics of users' MNO selection are based on the assumption that each user wants a specific service (or application) requiring some target QoS level. Then, each user's utility function becomes a step function with a step at QoS level. In other words, if a QoS level is higher than $\alpha$, then the utility from the service is equal to $\alpha$. Otherwise, the utility is zero. Therefore, a user with user type $\alpha$ subscribes MNO $i$'s communication service only if both $\alpha \geq p_i$ and $\alpha \leq q_i$ are satisfied.
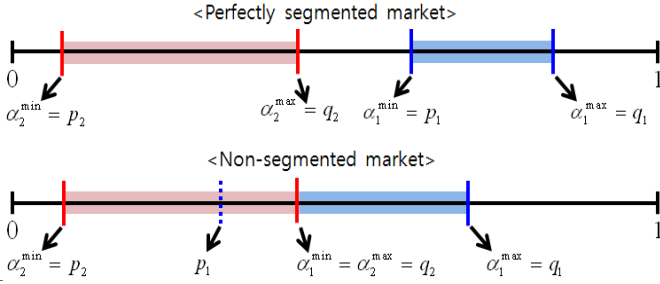
Fig. 3. User type intervals of the users subscribing each MNO's service.



Fig. 4. Two-stage Cournot and Bertrand competition between two MNOs.

reference capacity, which makes the QoS level equal to zero when all users access one of the MNOs. $k_1$ and $k_2$ are normalized values by the reference capacity. In other words, if $k_1 = 1$ ($k_2 = 1$) and $d_1 = M$ ($d_2 = M$), then the QoS level is $q_1 = 0$ ($q_2 = 0$). The user demand is expressed as follows:

$$d_1 = M \int_{\alpha_1^{\min}}^{\alpha_1^{\max}} f(\alpha)\, d\alpha, \quad d_2 = M \int_{\alpha_2^{\min}}^{\alpha_2^{\max}} f(\alpha)\, d\alpha, \quad (2)$$

where $\alpha_1^{\min}$, $\alpha_1^{\max}$, $\alpha_2^{\min}$ and $\alpha_2^{\max}$ denote the minimum and the maximum values of $\alpha$ among the users accessing the first and the second MNO, respectively. $f(\alpha)$ denotes a probability density function of $\alpha$. Equation (2) is a demand function derived from integration of the willingness-to-pay distribution.

Figure 3 illustrates a perfectly segmented market and non-segmented market. Assume $p_1 > p_2$. In the perfectly segmented market ($p_1 \geq q_2$), the values of $\alpha_1^{\min}$ and $\alpha_2^{\min}$ are determined by PC, and $\alpha_1^{\max}$ and $\alpha_2^{\max}$ are determined by QC. On the other hand, in the non-segmented duopoly market ($p_1 < q_2$), $\alpha_1^{\min}$ is determined by $\alpha_2^{\max}$. This is because if there are users whose PC and QC are satisfied by both MNOs[3], then the users whose user types are within $[p_1, q_2]$ will access the second MNO with the lower price, $p_2$.

## III. MOBILE NETWORK OPERATOR'S OPTIMIZATION PROBLEM

Figure 4 explains our two-stage model. In the Cournot stage, MNOs decide their capacity considering the investment cost. Each MNO cannot change its network capacity in the short-term after observing a competitor's network investment. Thus, this capacity competition can be modeled as a simultaneous game. Hereafter, let $i$ denote the decision maker index and $j$ denote the competitor's. Then, we formulate the optimization problem of MNO $i$ in the Cournot stage as follows:

$$\max_{k_i \geq 0} \quad f_i^R(k_i, k_j) - f_i^C(k_i), \quad (3)$$

where $f_i^R(\cdot)$ and $f_i^C(\cdot)$ denote revenue and cost functions of MNO $i$. Note that the MNO's revenue depends not only on its own capacity, but also on its competitor's. The revenue function is determined by the result of the Bertrand stage.

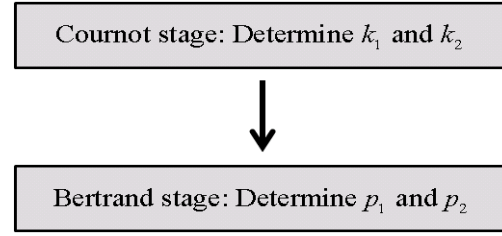[3]For the users in $[p_1, q_2]$, both PC and QC are satisfied by two MNOs.

In the Bertrand stage, MNOs compete with each other by controlling their prices for the given capacity determined in the Cournot stage. Here the price includes the subsidy. Thus high price implies low subsidy, and low price implies high subsidy. Each MNO can change its price repeatedly after observing a competitor's. Thus, this price competition can be modeled as an infinite sequential game. We exclude the case of pricing equal to that of the competitor. Then, we formulate the optimization problem of MNO $i$ in the Bertrand stage, which is divided into two cases: Using either a higher or lower price than the competitor's.

- *Using a lower price (higher subsidy), $p_i \leq p_j$:*

$$\max_{0 \leq p_i \leq 1} \quad p_i d_i$$

$$\text{s.t.} \quad d_i = M \int_{p_i}^{1 - \frac{d_i}{k_i M}} f(\alpha)\, d\alpha.$$

In this case, the number of users for MNO $i$ is independent of the competitor's price, which is just like the monopoly led by the low pricing MNO. Thus, the upper and lower limits of the integral in the constraint are replaced by $\alpha_i^{\max} = q_i = 1 - d_i/k_i M$ and $\alpha_i^{\min} = p_i$ as in Figure 3.

- *Using a higher price (lower subsidy), $p_i > p_j$:*

$$\max_{0 \leq p_i \leq 1} \quad p_i d_i$$

$$\text{s.t.} \quad d_i = M \int_{\max\left\{p_i, 1 - \frac{d_j}{k_j M}\right\}}^{1 - \frac{d_i}{k_i M}} f(\alpha)\, d\alpha.$$

In this case, the competitor affects the number of the users of MNO $i$. If the competitor guarantees PC and QC of a user, then the user will access the competitor network. Noting that $\alpha_i^{\max}$ is determined by the QC, the upper limit of the integral is given by $\alpha_i^{\max} = q_i = 1 - d_i/k_i M$. On the other hand, $\alpha_i^{\min}$ is determined by $\max\{p_i, q_j\}$ as explained in Figure 3.

## IV. BERTRAND STAGE: PRICE (SUBSIDY) COMPETITION

A common method for analyzing a multi-stage game is the backward induction method. This method is used to find the equilibrium that represents a Nash equilibrium in every stage (or subgame) of the original game. We start with the Bertrand stage. The capacities $k_i$ and $k_j$ are assumed to be given, and will be optimized in the next section describing

the Cournot stage. Hereafter, we assume that the user type $\alpha$ is uniformly distributed.[4]

### A. Price War with a Long Jump

We derive the optimal price of MNO $i$, which is summarized in the following lemmas:

**Lemma 1**: *In the case that the MNO's price $p_i$ is lower than its competitor's price $p_j$, the optimal solution $p_i^L$ is:*

$$p_i^L = \begin{cases} \frac{1}{2} & \text{if } p_j > \frac{1}{2} \\ p_j - \varepsilon & \text{if } p_j \leq \frac{1}{2} \end{cases}, \quad (4)$$

*where $\varepsilon$ is a minimum unit of price level changes and very small positive value.*

**Proof**: Under the assumption that $\alpha$ is uniformly distributed, we get the following equation from the constraint of the optimization problem for the lower price case:

$$d_i = M \int_{p_i}^{1 - \frac{d_i}{k_i M}} f(\alpha)\, d\alpha = M \left( 1 - \frac{d_i}{k_i M} - p_i \right). \quad (5)$$

We calculate $d_i$ and the objective function $p_i d_i$ from Equation (5) as follows:

$$d_i = \frac{k_i(1 - p_i)}{k_i + 1} M, \quad p_i d_i = \frac{k_i p_i (1 - p_i)}{k_i + 1} M. \quad (6)$$

The objective function is a quadratic function whose maximum is at $p_i = 1/2$. Therefore, if $p_j > 1/2$, then the optimal solution will be $p_i^L = 1/2$. Otherwise, the optimal solution will be $p_i^L = p_j - \varepsilon$. ∎

**Lemma 2**: *In the case that the MNO's price $p_i$ is higher than its competitor's price $p_j$, the optimal solution $p_i^H$ is:*

$$p_i^H = \begin{cases} \frac{k_j + p_j}{k_j + 1} & \text{if } p_j \geq \frac{1 - k_j}{2} \\ \frac{1}{2} & \text{if } p_j < \frac{1 - k_j}{2} \end{cases}. \quad (7)$$

**Proof**: Under the assumption that $\alpha$ is uniformly distributed, we get the following equation from the constraint of the optimization problem for the higher price case:

$$\begin{aligned} d_i &= M \int_{\max\left\{ p_i, 1 - \frac{d_j}{k_j M} \right\}}^{1 - \frac{d_i}{k_i M}} f(\alpha)\, d\alpha \\ &= M \left( 1 - \frac{d_i}{k_i M} - \max\left\{ p_i, 1 - \frac{d_j}{k_j M} \right\} \right). \end{aligned} \quad (8)$$

We calculate $d_i$ from Equation (8) as follows:

$$d_i = \min \left\{ \frac{k_i(1 - p_i)}{k_i + 1} M, \frac{k_i d_j}{(k_i + 1)k_j} \right\}. \quad (9)$$

Then, the objective function is:

$$\begin{aligned} p_i d_i &= \min \left\{ \frac{k_i p_i (1 - p_i)}{k_i + 1} M, \frac{k_i p_i d_j}{(k_i + 1)k_j} \right\} \\ &= \min \left\{ \frac{k_i p_i (1 - p_i)}{k_i + 1} M, \frac{k_i p_i (1 - p_j)}{(k_i + 1)(k_j + 1)} M \right\} \end{aligned} \quad (10)$$

[4]The assumption that the user type is uniformly distributed was also used in [1] and [14].

The second equality in Equation (10) holds by Equation (6). In the minimum operator of the objective function, the left side is a quadratic function whose maximum is at $p_i = 1/2$, and the right side is a linear function whose slope is $k_i(1 - p_j)M/((k_i + 1)(k_j + 1))$. Therefore, if $p_j < (1 - k_j)/2$, then the optimal solution is $p_i^H = 1/2$, which is the apex of the quadratic function. Otherwise, the optimal solution is $p_i^H = (k_j + p_j)/(k_j + 1)$, which is the intersection of the quadratic and linear functions. ∎

From Lemmas 1 and 2, we derive the MNO's best response function (optimal strategy) in the duopoly competition.

**Lemma 3**: *Given the competitor's price $p_j$, the MNO's best response function $p_i^*$ is*

- *Case 1 ($k_j < 1$):*

$$p_i^* = \begin{cases} \frac{1}{2} & \text{if } p_j > \frac{1}{2} \text{ or } 0 \leq p_j < \frac{1 - k_j}{2} \\ p_j - \varepsilon & \text{if } \frac{1}{k_j + 2} < p_j \leq \frac{1}{2} \\ \frac{k_j + p_j}{k_j + 1} & \text{if } \frac{1 - k_j}{2} \leq p_j \leq \frac{1}{k_j + 2} \end{cases} \quad (11)$$

- *Case 2 ($k_j \geq 1$):*

$$p_i^* = \begin{cases} \frac{1}{2} & \text{if } p_j > \frac{1}{2} \\ p_j - \varepsilon & \text{if } \frac{1}{k_j + 2} < p_j \leq \frac{1}{2} \\ \frac{k_j + p_j}{k_j + 1} & \text{if } 0 \leq p_j \leq \frac{1}{k_j + 2} \end{cases}. \quad (12)$$

**Proof**: Using the results of Lemmas 1 and 2, we calculate the optimal values for the higher and lower price cases as follows:

$$p_i^L d_i^L = \begin{cases} \frac{k_i}{4(k_i + 1)} M & \text{if } p_j > \frac{1}{2} \\ \frac{k_i(p_j - \varepsilon)(1 - p_j + \varepsilon)}{k_i + 1} M \approx \frac{k_i p_j (1 - p_j)}{k_i + 1} M & \text{if } p_j \leq \frac{1}{2} \end{cases},$$

$$p_i^H d_i^H = \begin{cases} \frac{k_i}{4(k_i + 1)} M & \text{if } p_j \geq \frac{1 - k_j}{2} \\ \frac{k_i(k_j + p_j)(1 - p_j)}{(k_i + 1)(k_j + 1)^2} M & \text{if } p_j < \frac{1 - k_j}{2} \end{cases}.$$

Above all, we consider Case 1 ($k_j < 1$). In this case, we calculate the best response function of MNO $i$ as follows:

- If $p_j > 1/2$: To compare the optimal values of the higher and lower price cases, we calculate the following:

$$\begin{aligned} p_i^L d_i^L - p_i^H d_i^H &= \frac{k_i M}{4(k_i + 1)} - \frac{k_i(k_j + p_j)(1 - p_j)M}{(k_i + 1)(k_j + 1)^2} \\ &= \frac{k_i(k_i + 2p_j - 1)^2 M}{4(k_i + 1)(k_j + 1)^2}. \end{aligned}$$

This value is positive because we consider the duopoly market (i.e., $k_i$ and $k_j$ are positive) and assume $p_j > 1/2$. Therefore, the best response function is $p_i^* = p_i^L = 1/2$.

- If $(1 - k_j)/2 \leq p_j \leq 1/2$: To compare the optimal values of the higher and lower price cases, we calculate

the following:

$$p_i^L d_i^L - p_i^H d_i^H$$
$$= \frac{k_i p_j (1-p_j) M}{k_i + 1} - \frac{k_i (k_j + p_j)(1-p_j) M}{(k_i + 1)(k_j + 1)^2}$$
$$= \frac{k_i k_j (k_j + 2)(1-p_j) M}{(k_i + 1)(k_j + 1)^2}\left(p_j - \frac{1}{k_j + 2}\right).$$

All values except $p_j - 1/(k_j + 2)$ are positive because we consider the duopoly market (i.e., $k_i$ and $k_j$ are positive) and assume $(1 - k_j)/2 \leq p_j \leq 1/2$. Therefore, if $p_j > 1/(k_j + 2)$, then the best response function is $p_i^* = p_i^L = p_j - \varepsilon$. Otherwise, the best response function is $p_i^* = p_i^H = (k_j + p_j)/(k_j + 1)$.

- If $0 \leq p_j < (1 - k_j)/2$: To compare the optimal values of the higher and lower price cases, we calculate the following:

$$p_i^H d_i^H - p_i^L d_i^L = \frac{k_i M}{4(k_i + 1)} - \frac{k_i p_j (1-p_j) M}{k_i + 1}$$
$$= \frac{k_i M}{k_i + 1}\left(\frac{1}{4} - p_j(1-p_j)\right).$$

This value is positive under the assumption $0 \leq p_j < (1 - k_j)/2$. Therefore, the best response function is $p_i^* = p_i^H = 1/2$.

Noting that the intervals $[0, (1 - k_j)/2)$ and $[(1 - k_j)/2, 1/(k_j + 2)]$ in Case 1 are merged into an interval $[0, 1/(k_j + 2)]$ in Case 2, we can derive the MNO's best response function in Case 2 similarly. ∎

Using Lemma 3, we describe two MNOs dynamics below.

**Proposition 1**: *In the duopoly competition of two MNOs, there is no pure Nash equilibrium, and price levels increase and decrease periodically.*[5]

**Proof**: Suppose for a contradiction that a pure Nash equilibrium point $(p_i^{NE}, p_j^{NE})$ exists. We first consider Case 1 ($k_j < 1$) as follows:

- If $p_j^{NE} > 1/2$, then $p_i^{NE}$ should be $p_i^{NE} = 1/2$. Then, $p_j^{NE}$ should be $p_j^{NE} = 1/2 - \varepsilon$, which contradicts the assumption that $p_j^{NE} > 1/2$.
- If $1/(k_j + 2) < p_j^{NE} \leq 1/2$, then $p_i^{NE}$ should be $p_i^{NE} = p_j^{NE} - \varepsilon < 1/2$. Then, $p_j^{NE}$ should satisfy at least one of the following equations:

$$p_j^{NE} = p_i^{NE} - \varepsilon = p_j^{NE} - 2\varepsilon, \tag{13}$$

$$p_j^{NE} = \frac{k_i + p_i^{NE}}{k_i + 1} = \frac{k_i + p_j^{NE} - \varepsilon}{k_i + 1}. \tag{14}$$

Obviously, Equation (13) has a contradiction. From Equation (14), we find that $p_j^{NE}$ is $p_j^{NE} = 1 - \varepsilon/k_i \approx 1$, which contradicts the assumption that $p_j^{NE} \leq 1/2$.

---

[5]Even though there is no Nash equilibrium in pure strategies, there can be a Nash equilibrium in mixed strategies. This means that the MNOs can randomize their service prices. However, in the sequential game that each MNO can change its price after observing a competitor's, the best response function-based strategy will be used rather than the random strategy. This is the reason that we focus on the price dynamics.

- If $(1 - k_j)/2 \leq p_j^{NE} \leq 1/(k_j + 2)$, then $p_j^{NE}$ should be $p_i^{NE} = (k_j + p_j^{NE})/(k_j + 1)$. Then, $p_j^{NE}$ should satisfy at least one of the following equations:

$$p_j^{NE} = \frac{1}{2}, \tag{15}$$

$$p_j^{NE} = p_i^{NE} - \varepsilon = \frac{k_j + p_j^{NE}}{k_j + 1} - \varepsilon, \tag{16}$$

$$p_j^{NE} = \frac{k_i + p_i^{NE}}{k_i + 1} = \frac{k_i + \frac{k_j + p_j^{NE}}{k_j + 1}}{k_i + 1}. \tag{17}$$

Based on the assumption that $p_j^{NE} \leq 1/(k_j + 2)$, Equation (15) means that $k_j = 0$ and $p_i^{NE} = p_j^{NE} = 1/2$. This has a contradiction because $p_i^{NE} = 1/2$ and $p_j^{NE} = 1/2$ are not the best response to each other. Using Equation (16), we calculate that $p_j^{NE}$ is $p_j^{NE} = 1 - \varepsilon - \varepsilon/k_j \approx 1$, which contradicts the assumption that $p_j^{NE} \leq 1/(k_j + 2)$. Using Equation (17), we calculate that $p_j^{NE}$ is $p_j^{NE} = 1$, which contradicts the assumption that $p_j^{NE} \leq 1/(k_j + 2)$.

- If $0 \leq p_j^{NE} < (1 - k_j)/2$, then $p_i^{NE}$ should be $p_i^{NE} = 1/2$. This means that $p_j^{NE}$ should be $p_j^{NE} = 1/2 - \varepsilon \approx 1/2$, which contradicts the assumption that $p_j^{NE} < (1 - k_j)/2$.

Therefore, we conclude that there is no pure Nash equilibrium point in Case 1. The only difference between Case 1 and Case 2 ($k_j \geq 1$) is that the intervals $[0, (1 - k_j)/2)$ and $[(1 - k_j)/2, 1/(k_j + 2)]$ in Case 1 are merged in the interval $[0, 1/(k_j + 2)]$ in Case 2. Therefore, we can use similar proof for Case 2 and conclude that there is no pure Nash equilibrium point in Case 2. ∎

Figure 5 shows the MNOs' best response functions. In the figure, we assume the symmetric capacity ($k_1 = k_2 = k$). We see how each MNO sequentially decreases its price to a little lower level than its competitor when the competitor's price is within $(1/(k + 2), 1/2]$. This is a *price war* [7], [8]. After that, if the competitor's price is less than or equal to $1/(k+2)$, then one MNO will increase its price to $(k + 1)/(k + 2)$. This is a *long jump*. Then, the competitor sets its price to $1/2$. This situation is repeated periodically and there is no equilibrium point. We call this *price war with a long jump*. We plot Figure 6 to illustrate the process of the price war with a long jump. In the real world, this kind of price dynamics tends to occur by change of the subsidy amount (see Figure 2).

### B. Regulation for Convergence

A price war with a long jump is not desirable because of instability and inefficiency. We suggest a simple regulation that leads to an equilibrium point of price levels, which is Pareto-optimal. More details are contained in Proposition 2.

**Lemma 4**: *A regulation that limits the number of price level changes by two MNOs makes the price levels converge to*
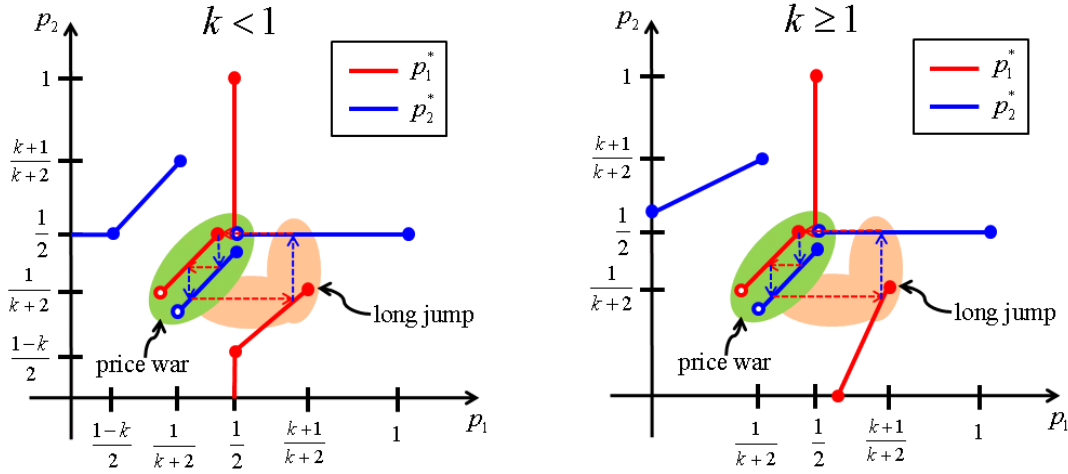
Fig. 5. Best response functions of two MNOs. We assume that the MNOs' capacities are equal ($k_1 = k_2 = k$).
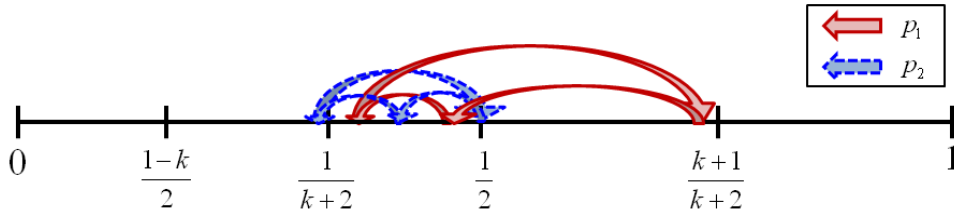


Fig. 6. Price war with a long jump ($k_1 = k_2 = k$).

an equilibrium point $(p_i^E, p_j^E)$ as follows:

$$(p_i^E, p_j^E) = \begin{cases} \left(\frac{1}{k_i+2}, \frac{k_i+1}{k_i+2}\right) & \text{if } k_i < 2k_j \\ \left(\frac{1}{k_i+2}, \frac{k_i+1}{k_i+2}\right) \text{ or } \left(\frac{2k_j+1}{2(k_j+1)}, \frac{1}{2}\right) & \text{if } k_i = 2k_j \\ \left(\frac{2k_j+1}{2(k_j+1)}, \frac{1}{2}\right) & \text{if } k_i > 2k_j \end{cases}$$

where the last opportunity for a price level change is given to MNO $j$.

**Proof**: When we use the regulation, two MNOs dynamics is modeled as a finite sequential game and we can calculate the equilibrium point using the backward induction method. We first consider Case 1 ($k_j < 1$). Let $t$ denote the last stage of the price level change. Then, MNO $i$ will choose the best strategy at the $(t-1)$ stage (i.e., its last choice stage) in order to maximize its revenue. To calculate the best strategy for MNO $i$, we divide the strategy set of MNO $i$ into four disjoint subsets. Let $p_i^*(t-1)$ and $p_j^*(t)$ denote the optimal strategy of MNO $i$ at the $t-1$ stage and the optimal strategy of MNO $j$ at the last stage in each subset. Using Lemma 3, we calculate $p_i^*(t-1)$ in each subset. Then, from Equations (6) and (10), we calculate the revenue of MNO $i$ $r_i^*(t)$ as follows:

*Strategy 1* ($p_i(t-1) > 1/2$):

$$p_j^*(t) = \frac{1}{2} \rightarrow p_i^*(t-1) = \frac{k_j + \frac{1}{2}}{k_j+1} = \frac{2k_j+1}{2(k_j+1)},$$

$$r_i^*(t) = \frac{k_i p_i^*(t)(1-p_i^*(t))}{k_i+1} M = \frac{k_i(2k_j+1)}{4(k_i+1)(k_j+1)^2} M.$$

*Strategy 2* ($1/(k_i+2) < p_i(t-1) \leq 1/2$):

$$p_j^*(t) = p_i(t-1) - \varepsilon \rightarrow p_i^*(t-1) = \frac{1}{2},$$

$$r_i^*(t) = \frac{k_i p_i^*(t-1)(1-p_j^*(t))}{(k_i+1)(k_j+1)} M = \frac{k_i(1+2\varepsilon)}{4(k_i+1)(k_j+1)} M.$$

*Strategy 3* ($(1-k_i)/2 \leq p_i(t-1) \leq 1/(k_i+2)$):

$$p_j^*(t) = \frac{k_i + p_i(t-1)}{k_i+1} \rightarrow p_i^*(t-1) = \frac{1}{k_i+2},$$

$$r_i^*(t) = \frac{k_i p_i^*(t)(1-p_i^*(t))}{k_i+1} M = \frac{k_i}{(k_i+2)^2} M.$$

*Strategy 4* ($0 \leq p_i(t-1) < (1-k_i)/2$):

$$p_j^*(t) = \frac{1}{2} \rightarrow p_i^*(t-1) = \frac{1-k_i}{2} - \varepsilon,$$

$$r_i^*(t) = \frac{k_i p_i^*(t)(1-p_i^*(t))}{k_i+1} M = \frac{k_i\left(1-(k_i+\varepsilon)^2\right)}{4(k_i+1)} M.$$

From these equations, we prove that some strategies are strictly dominated as follows:

- Strategy 2 is strictly dominated by Strategy 1:

$$\frac{k_i(2k_j+1)}{4(k_i+1)(k_j+1)^2} M - \frac{k_i(1+2\varepsilon)}{4(k_i+1)(k_j+1)} M$$

$$= \frac{k_i}{4(k_i+1)(k_j+1)}\left(\frac{k_j}{k_j+1} - 2\varepsilon\right) M > 0.$$

- Strategy 4 is strictly dominated by Strategy 3:

$$\frac{k_i}{(k_i+2)^2}M - \frac{k_i\left(1-(k_+\varepsilon)^2\right)}{4(k_i+1)}M$$
$$> \frac{k_i}{(k_i+2)^2}M - \frac{k_i\left(1-k_i^2\right)}{4(k_i+1)}M$$
$$= \frac{k_i^3(k_i+3)}{4(k_i+2)^2}M > 0.$$

Therefore, the MNO $i$ never chooses Strategies 2 and 4. To finish this proof, we compare Strategies 1 and 3 as follows:

$$\frac{k_i(2k_j+1)}{4(k_i+1)(k_j+1)^2}M - \frac{k_i}{(k_i+2)^2}M$$
$$= \frac{k_i(k_i+2(k_i+1)k_j)(k_i-2k_j)}{4(k_i+1)(k_i+2)^2(k_j+1)^2}M.$$

If $k_i < 2k_j$, then MNO $i$ will set its price to $1/(k_i+2)$ and MNO $j$ will set its price to $(k_i+1)/(k_i+2)$. Therefore, in this case, the equilibrium point is $(p_i^E, p_j^E) = (1/(k_i+2), (k_i+1)/(k_i+2))$. Likewise, if $k_i > 2k_j$, then the equilibrium point is $(p_i^E, p_j^E) = ((2k_j+1)/(2(k_j+1)), 1/2)$. Note that, if $k_i = 2k_j$, then both equilibriums are possible. The only difference between Case 1 and Case 2 is that the intervals $[0, (1-k_j)/2)$ and $[(1-k_j)/2, 1/(k_j+2)]$ in Case 1 are merged into the interval $[0, 1/(k_j+2)]$ in Case 2. Therefore, we can use a similar proof for Case 2 and conclude that the equilibrium in Case 2 is same to that in Case 1. ∎

To verify the efficiency of the regulation, we need to check the Pareto-optimality of the equilibrium point. A sufficient condition for Pareto-optimality is given in the following lemma:

**Lemma 5**: *Let subscripts $l$ and $h$ denote MNOs whose prices are lower and higher than that of their competitor, respectively. If $p_l \leq 1/2$, $p_h \geq 1/2$ and $p_h \geq (k_l+p_l)/(k_l+1)$, then $(p_l, p_h)$ and $(p_h, p_l)$ are Pareto-optimal.*

**Proof**: If $p_l < 1/2$, then MNO $l$ can increase its revenue by increasing its price toward $1/2$. However, it always makes MNO $h$'s revenue decrease. If $p_l = 1/2$, then MNO $l$ cannot increase its revenue. Therefore, MNO $l$ cannot increase its revenue without decreasing the revenue of MNO $h$. Likewise, MNO $h$ cannot increase its revenue without decreasing the revenue of MNO $l$ because the only method to increase the revenue of MNO $h$ in the perfectly segmented condition ($p_h \geq (k_l+p_l)/(k_l+1)$) is to decrease $p_l$ and $p_h$ simultaneously. Therefore, the point that satisfies the conditions in Lemma 5 is Pareto-optimal. ∎

Lemma 5 indicates that the MNOs' prices should be sufficiently separated (i.e., perfectly segmented market) for Pareto-optimality. The equilibrium point under our suggested regulation (Lemma 4) satisfies the conditions in Lemma 5. Therefore, the equilibrium point is Pareto-optimal. This finding is summarized below.

**Proposition 2**: *A regulation that limits the number of price level changes by two MNOs makes the price levels converge to an equilibrium point that is Pareto-optimal.*

The price war with a long jump occurs due to the MNOs' short-sighted way of thinking. However, the MNOs cannot use a myopic strategy under the price regulation. This enables us to infer that the price regulation leads price levels to a Pareto-optimal point.

In an opposite view, the price regulation is not realistic because upperbound of the service price is actually set up but few regulations can be observed over the number of pricing level changes. However, the price regulation can be implemented by restricting subsidization. In other words, notifying MNOs that subsidization will be banned after a certain time is equal to the regulation that limits the number of price level changes. For instance, Korea Communications Commission (KCC) actually prohibited subsidization by MNOs from 2003 to 2008. This kind of subsidy regulation also can be observed in Finland [18]. Another way that brings the same effect of the price regulation is to regulate the time interval between price level changes. If MNOs cannot change their service price or subsidy levels for a long time, then they will use far-sighted strategies, which leads to the equilibrium point described in Lemma 4.

## V. COURNOT STAGE: CAPACITY COMPETITION

We combine the result of the Bertrand stage with that of the Cournot stage. That is, using the results of Section IV, we rewrite the optimization problem (Equation (3)) of the Cournot stage and solve it. The main motivation of this section is to understand the competitive actions of each MNO, and to derive the optimal response of the regulator.

### A. Characteristics of Communications Service Duopoly Market

Without loss of generality, we assume that the last opportunity for a price level change is given to MNO $j$. Using the revenue equations (Equations (6) and (10)), we calculate the revenue function $f_i^R(k_i, k_j)$ at the equilibrium price of the Bertrand stage (Lemma 4). We use the linearly increasing cost function as in [16]. That is, $f_i^C(k_i) = \gamma M k_i$, where $\gamma$ is the unit cost per capacity. Then, we can rewrite the optimization problem (Equation (3)) of the Cournot stage, which is divided into two cases.

- *Case 1 ($k_i \leq 2k_j$):*

$$\max_{k_i \geq 0} \quad \frac{Mk_i}{(k_i+2)^2} - \gamma M k_i, \tag{18}$$

$$\max_{k_j \geq 0} \quad \frac{M(k_i+1)}{(k_i+2)^2}\frac{k_j}{k_j+1} - \gamma M k_j. \tag{19}$$

- *Case 2 ($k_i \geq 2k_j$):*

$$\max_{k_i \geq 0} \quad \frac{M(2k_j+1)}{4(k_j+1)^2}\frac{k_i}{k_i+1} - \gamma M k_i, \tag{20}$$

$$\max_{k_j \geq 0} \quad \frac{M}{4}\frac{k_j}{k_j+1} - \gamma M k_j. \tag{21}$$

To solve these optimization problems, we need some mathematical knowledge given in the lemmas below.

**Lemma 6**: *Consider the following optimization problem:*

$$\max_{x \geq 0} \quad \frac{bx}{x + a} - cx, \tag{22}$$

*where a, b and c are positive. Then, the optimal solution $x^*$ is*

$$x^* = \max\left\{0, \sqrt{\frac{ab}{c}} - a\right\}. \tag{23}$$

**Proof**: We calculate the first and second order derivatives of the objective function as follows:

$$\left(\frac{bx}{x + a} - cx\right)' = \frac{ab}{(x + a)^2} - c, \tag{24}$$

$$\left(\frac{bx}{x + a} - cx\right)'' = \frac{-2ab(x + a)}{(x + a)^4}. \tag{25}$$

The problem is a convex optimization problem because the second order derivative is always negative in the feasible set. Therefore, using the first order condition, we calculate the optimal solution as follows:

$$x^* = \max\left\{0, \sqrt{\frac{ab}{c}} - a\right\}. \tag{26}$$

∎

**Lemma 7**: *Consider the following optimization problem:*

$$\max_{x \geq 0} \quad \frac{bx}{(x + a)^2} - cx, \tag{27}$$

*where a, b and c are positive. Then, the optimal solution $x^*$ is*

$$x^* = \max\left\{0, -a + \sqrt[3]{\frac{ab}{c} + \sqrt{\frac{a^2b^2}{c^2} + \frac{b^3}{27c^3}}} + \sqrt[3]{\frac{ab}{c} - \sqrt{\frac{a^2b^2}{c^2} + \frac{b^3}{27c^3}}}\right\}. \tag{28}$$

**Proof**: We calculate the first and second order derivatives of the objective function as follows:

$$\left(\frac{bx}{(x + a)^2} - cx\right)' = \frac{-bx + ab}{(x + a)^3} - c, \tag{29}$$

$$\left(\frac{bx}{(x + a)^2} - cx\right)'' = \frac{2bx - 4ab}{(x + a)^4}. \tag{30}$$

The objective function is partially concave because the second order derivative is positive when $x > 2a$. However, from the first order derivative, we know that the objective function decreases as $x$ increases when $x > a$. Therefore, we only consider $0 \leq x \leq a$ as the feasible set, and the optimization problem becomes a convex optimization

problem in the set. Then, using the first order condition and the root formula of the third order equation, we calculate the optimal solution $x^*$. ∎

Using Lemmas 6 and 7, we calculate the MNOs' optimal solutions $k_i^*$ and $k_j^*$ as follows:

- *Case 1 $(k_i \leq 2k_j)$:*

$$k_i^* = \max\left\{0, -2 + \sqrt[3]{\frac{2}{\gamma} + \sqrt{\frac{4}{\gamma^2} + \frac{1}{27\gamma^3}}}\right.$$
$$\left. + \sqrt[3]{\frac{2}{\gamma} - \sqrt{\frac{4}{\gamma^2} + \frac{1}{27\gamma^3}}}\right\}, \tag{31}$$

$$k_j^* = \max\left\{0, \sqrt{\frac{k_i + 1}{(k_i + 2)^2 \gamma}} - 1\right\}. \tag{32}$$

- *Case 2 $(k_i \geq 2k_j)$:*

$$k_i^* = \max\left\{0, \sqrt{\frac{2k_j + 1}{4(k_j + 1)^2 \gamma}} - 1\right\}, \tag{33}$$

$$k_j^* = \max\left\{0, \sqrt{\frac{1}{4\gamma} - 1.}\right\} \tag{34}$$

From this result, we find the equilibrium of the original two-stage game as follows:

**Proposition 3**: *Under the regulation that limits the price level changes by MNOs, if the unit cost per capacity $\gamma$ satisfies the following inequality,*

$$F(\gamma) = \sqrt[3]{\frac{2}{\gamma} + \sqrt{\frac{4}{\gamma^2} + \frac{1}{27\gamma^3}}} + \sqrt[3]{\frac{2}{\gamma} - \sqrt{\frac{4}{\gamma^2} + \frac{1}{27\gamma^3}}} > 2, \tag{35}$$
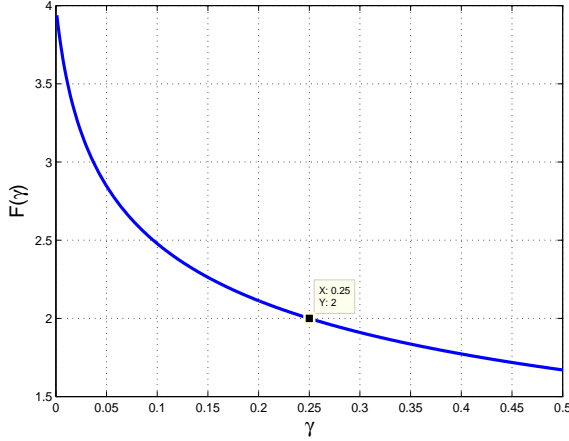
*then there is an equilibrium point $(k_i^E, k_j^E, p_i^E, p_j^E)$:*

$$\begin{aligned} &\left(k_i^E, k_j^E, p_i^E, p_j^E\right) \\ &= \left(k_i^E, \sqrt{\frac{k_i^E + 1}{(k_i^E + 2)^2 \gamma}} - 1, \frac{1}{k_i^E + 2}, \frac{k_i^E + 1}{k_i^E + 2}\right) \\ &= \left(F(\gamma) - 2, \sqrt{\frac{F(\gamma) - 1}{F(\gamma)^2 \gamma}} - 1, \frac{1}{F(\gamma)}, 1 - \frac{1}{F(\gamma)}\right), \end{aligned} \tag{36}$$

*which is Pareto-optimal.*

**Proof**: In Case 2 $(k_i^* \geq 2k_j^*)$, to be an equilibrium point, the optimal solution should satisfy the following equation:

$$\begin{aligned} k_i^* &= \max\left\{0, \sqrt{\frac{2k_j^* + 1}{4(k_j^* + 1)^2 \gamma}} - 1\right\} \\ &\leq \max\left\{0, \sqrt{\frac{k_j^{*2} + 2k_j^* + 1}{4(k_j^* + 1)^2 \gamma}} - 1\right\} \\ &= \max\left\{0, \sqrt{\frac{1}{4\gamma}} - 1\right\} = k_j^*, \end{aligned} \tag{37}$$

Fig. 7. Feasibility function $F(\gamma)$.



Fig. 8. Equilibrium capacities $k_i^E$ and $k_j^E$.

which contradicts the assumption $k_i^* \geq 2k_j^*$ because $k_i^*$ and $k_j^*$ are non-zero. Therefore, there is no equilibrium point in Case 2.

In Case 1 $(k_i^* \leq 2k_j^*)$, to be an equilibrium point, the optimal solution should satisfy the following equation:

$$2k_j^* - k_i^* = 2\sqrt{\frac{k_i^* + 1}{(k_i^* + 2)^2 \gamma}} - (k_i^* + 2) \geq 0$$

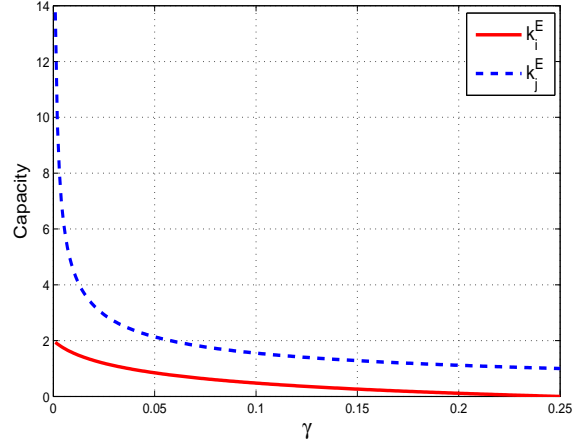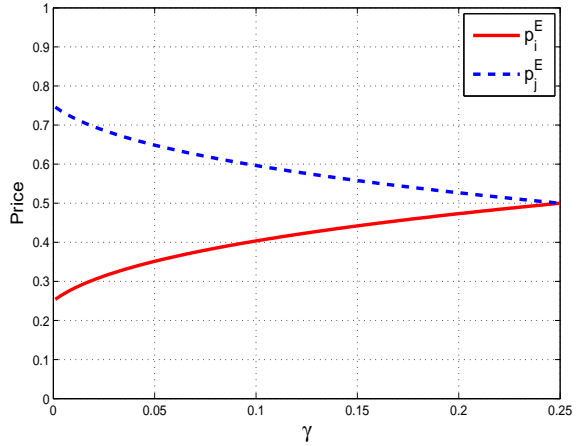$$\Rightarrow 2\sqrt{\frac{k_i^* + 1}{(k_i^* + 2)^2 \gamma}} \geq k_i^* + 2. \qquad (38)$$

The left and right hand side equations of the last inequality are positive. Thus, we compare the squares of them as follows:

$$\left(2\sqrt{\frac{k_i^* + 1}{(k_i^* + 2)^2 \gamma}}\right)^2 - (k_i^* + 2)^2$$

$$= \frac{4k_i^* + 4 - (k_i^* + 2)^4 \gamma}{(k_i^* + 2)^2 \gamma}$$

$$= \frac{4k_i^* + 4 - (k_i^* + 2)(-k_i^* + 2)}{(k_i^* + 2)^2 \gamma}$$

$$= \frac{k_i^*(k_i^* + 4)}{(k_i^* + 2)^2 \gamma} \geq 0.$$

Note that the second equality holds because we use an equation $(k_i^* + 2)^3 \gamma = -k_i^* + 2$ from the first order condition of the optimization problem. From the above calculations, we conclude that if $k_i^* > 0$ (i.e., $F(\gamma) > 2$), then there would be an equilibrium. Using these results and Lemma 4, we can calculate the equilibrium as in Proposition 3. ∎

We call $F(\gamma)$ the *feasibility function* because we can discriminate between feasible and infeasible markets using this function.[6] If the unit cost $\gamma$ is expensive and does not satisfy Equation (35), then the market will be infeasible (i.e., market failure) because $k_i^E$ and $k_j^E$ are negative values.

[6]By a feasible market, we mean there is at least one operator for the market.



Fig. 9. Equilibrium prices $p_i^E$ and $p_j^E$.

Note the equilibrium point is a function of $k_i^E$ in Equation (36). This means MNO $i$ has *market power* even though the regulator gives the last opportunity for changing the price level to MNO $j$.

Figure 7 shows the feasibility function, where the unit cost $\gamma$ should be less than $0.25$ to avoid market failure. We plot the equilibrium point in Proposition 3 varying $\gamma \in [0, 0.25]$. Figures 8 and 9 show the result. We observe that $p_j^E$ is always higher than $p_i^E$, and MNO $j$ always invests more than MNO $i$. This means the users with high user type (i.e., high QoS requirement and high willingness-to-pay) are targeted by MNO $j$. On the other hand, MNO $i$ can make a profit by making a relatively small investment because its target users have low user types. An interesting observation is that the price gap between both MNOs decreases as the unit cost per capacity increases in Figure 9. This is because that the high cost makes both MNOs reduce their investment levels and concentrate on lucrative targets (i.e., the users whose user types are near $0.5$).
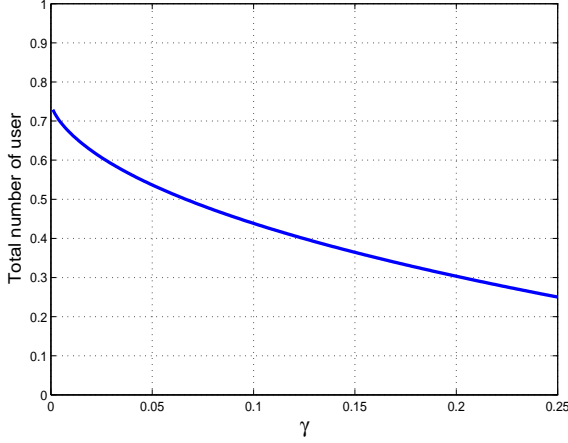
Fig. 10. Total number of users subscribing the communication service of either MNO $i$ or $j$. The value is divided by $M$.
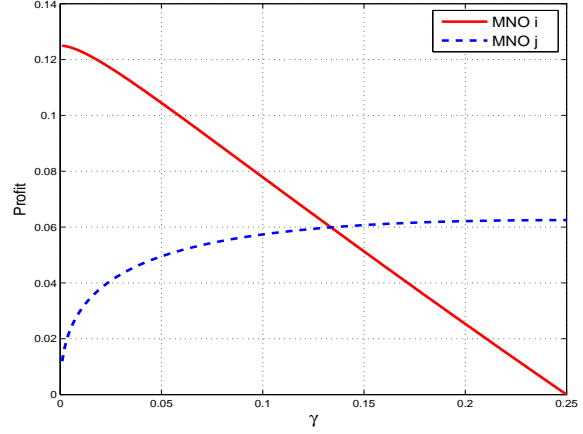


Fig. 11. Profits of two MNOs. The values are divided by $M$.

### B. Role of the Regulator

The regulator's key concern is to improve user welfare [19]. In this perspective, monitoring the total number of users who can enjoy the communication service is important. Another important thing is to ensure fairness between MNOs for fair competition because user welfare can decrease if there is a monopolist or a market-dominating enterprise [20]. The regulator can achieve his purpose by exacting *taxes* from the MNOs or giving *subsidies* to them. So far, we assume that the unit cost per capacity $\gamma$ is a given parameter. However, we can divide $\gamma$ into $\gamma_c$ and $\gamma_t$ (i.e., $\gamma = \gamma_c + \gamma_t$). The value $\gamma_c$ denotes the fixed cost, and $\gamma_t$ denotes the tax ($\gamma_t > 0$) or subsidy ($\gamma_t < 0$). Then, we suggest the regulator's optimal action corresponding to its objective as described below.

- *User welfare perspective*: We plot the total number of users subscribing the communication service of either MNO $i$ or $j$ in Figure 10. The figure shows that the total number of users decreases as $\gamma$ increases. Even though this result is predictable, the regulator can use it to forecast results of exacting taxes or giving subsidies. Using the fact that the market becomes a monopoly when $\gamma$ is equal to 0.25, an interesting point is that the total number of subscribers under the duopoly competition is always higher than that under the monopoly. This means that competition between MNOs can lead to increase of user welfare.

- *Fairness perspective*: We plot the profits of two MNOs in Figure 11. The figure shows that the profit of MNO $i$ decreases as $\gamma$ increases. Interestingly, the profit of MNO $j$ increases as $\gamma$ increases. This is because that MNO $i$ should reduce its investment amount when $\gamma$ is high, which gives MNO $j$ an opportunity to expand its service to the users of low user type who could not access MNO $j$'s network before. The regulator can adjust the fairness between the MNOs by exacting taxes or giving subsidies. In the figure, we observe that $\gamma$ should be about 0.13 to ensure the fairness.

## VI. Conclusions

MNOs tend to compete with each other changing their service prices by subsidization in the real world (see Figure 2). In this paper, to theoretically explain the price dynamics in the mobile communication service, we used a two-stage Cournot and Bertrand competition model that is well understood in microeconomics. The Cournot and Bertrand models are interlinked and we perform a joint optimization of network capacity and service price. Based on the game-theoretic approach, we show that there is a price war with a long jump. This price dynamics explains the subsidy dynamics in the real world. To avoid the instability and inefficiency, we propose a regulation that ensures an equilibrium point of price levels, which is Pareto-optimal. Based on our results in the Cournot stage, we describe characteristics of the duopoly market and suggest the regulator's optimal actions (exacting taxes) corresponding to the objectives such as user welfare maximization or fairness between MNOs. Although our analytic results are derived under some assumptions for mathematical tractability, it will provide good intuition for understanding the price dynamics and imposing regulations in the mobile communication service.

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